DALHOUSIE UNIVERSITY

Faculty of Science Course Syllabus
Department of Mathematics & Statistics
MATH 4500/5500 (Online)
Introduction to Harmonic Analysis
Winter 2021

Instructor(s): Professor Suresh Eswarathasan

Email: *sr766936@dal.ca* ("MATH 4500/5500" must be mentioned in the subject line to receive a reply. I can only respond to emails between 8am-5pm Monday-Friday; please give me at most 24 hours to respond)

Official Synchronous Schedule: MTTh, 8:35am-9:25am AST

Office Hours: On Zoom or Collaborate BB via appointment.

Lectures: *All lectures will be held over Zoom*, recorded, and uploaded onto BrightSpace. Lecture notes will also be uploaded at the end of the week.

Tutorials: No tutorials.

Course Description

Harmonic analysis is, roughly speaking, the quantitative study of functions on domains and similar objects such as measures or distributions. For simplicity, let us restrict to functions and consider the following two problems: P1) What is the most efficient way to decompose a function in a certain manner, or how does the size of it in one norm is related to the size in another? In fact, such problems are initially encountered in linear algebra. P2) Consider linear mappings that take a function as input and returns another as output, and understand how the size of the output (as quantified by various norms) relates to the size of the input. As in the first stated problem, one has already seen elementary forms of P2 when working with matrices. Our goal for the semester is to understand these two infinite-dimensional linear algebra-type problems via the help of Fourier analysis, complex analysis, and abstract algebra.

Harmonic analysis has long been intertwined with Fourier analysis, the study of how functions on symmetric spaces are decomposed into more symmetric objects (such group characters). Thanks to this and its connections with complex variables, harmonic analysis has played a pivotal role in number theory (to name one amongst many fields), particularly in our understanding of the *distribution of prime numbers* and multiplicative arithmetic functions like the *Möbius function*. Furthermore, the famous *Kakeya problem* in geometric measure theory has seen tremendous progress thanks to harmonic analysis, itself drawing ideas from arithmetic combinatorics and number theory.

Clearly, this area sits at the crossroads of a variety of fields in mathematics and our aim is to learn enough of the fundamentals to witness this in a single semester. Topics will include Fourier analysis on \R^n, Hausdorff/fractal dimension and measures, the Kakeya problem and its applications, and if time permits the connection between the Kakeya problem and local smoothing for wave equations

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Course Prerequisites
MATH 4010, MATH 3501 + 3502

Learning Objectives
Students will develop an understanding of various aspects of harmonic analysis on Euclidean spaces, including standard techniques surrounding Fourier transforms, singular integral operators, and Hausdorff dimension.

Course Materials

- Uploaded lectures notes. No course textbook will be used. Suggested references: T. Wolff "Harmonic Analysis"
- Course website on Brightspace is accessed through dal.brightspace.com

Course Assessment

For 4500 students:
85 % - weekly homework assignments
15 % - final report/presentation, depending on student's choice. Tentative due dates: April 7, 2021 if presentation is selected and April 15, 2021 if report is selected.

For 5500 students:
85 % - weekly homework assignments (with additional, graduate student-specific problems on each sheet)
15 % - take-home examination. Tentative due date: April 15, 2021

Conversion of numerical grades to Final Letter Grades follows the Dalhousie Common Grade Scale

A+ (90-100)	**B+** (77-79)	**C+** (65-69)	**D** (50-54)
A (85-89)	**B** (73-76)	**C** (60-64)	**F** (<50)
A- (80-84)	**B-** (70-72)	**C-** (55-59)	

Course Policies

- Any excuse must be valid as per the current Dalhousie University guidelines. Given the fluidity of the situation, particularly in light of the COVID-19 pandemic, the university may change these policies and therefore certain leniencies will be in place. Medical notes are not absolutely required for HW extension requests.
- Students are expected to use the Student Declaration of Absence form for missed quizzes or homework extension requests, and can be used as many times as needed.
- If a student misses a deadline, arrangements can be made (following one-on-one discussions) for later submissions or substitutions.

DALHOUSIE UNIVERSITY

- Regarding missed HWs, the instructor must be contacted and a one-on-one discussion with a legitimate excuse, as per Dalhousie University guidelines, must be held.
- Students are expected to work on their own and consult regularly with the instructor as they progress through the exercises.

Course Content (tentative)

- Chapter 1 = Convergence of Fourier series
- Chapter 2 = L^1 Fourier Transform
- Chapter 3 = Fourier Inversion
- Chapter 4 = L^p Theorems for Fourier Transforms
- Chapter 5 = Uncertainty Principle
- Chapter 6 = Calderon-Zygmund + Singular Integral Operators
- Chapter 7 = Almost Orthogonality
- Chapter 8 = Hausdorff Dimension + Measures